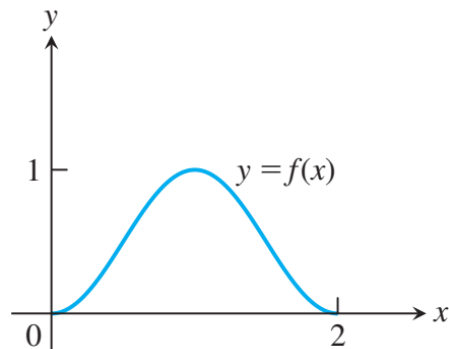# Exercise 57

The accompanying figure shows the graph of a function $f(x)$ with domain $[0, 2]$ and range $[0, 1]$. Find the domains and ranges of the following functions, and sketch their graphs.
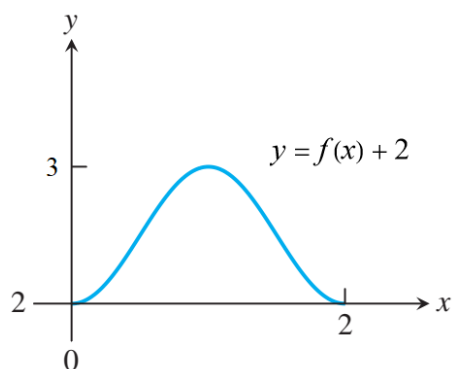

a. $f(x) + 2$ **b.** $f(x) - 1$

c. $2f(x)$ **d.** $-f(x)$

e. $f(x + 2)$ **f.** $f(x - 1)$

g. $f(-x)$ **h.** $-f(x + 1) + 1$

Solution

Part a.

$$f(x) + 2$$
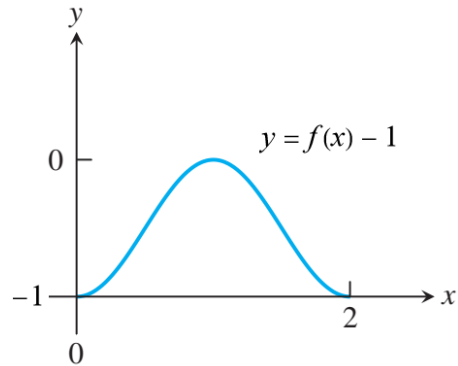
Adding 2 shifts the graph up by 2 units.

The domain of $f(x) + 2$ is $[0, 2]$, and the range of $f(x) + 2$ is $[2, 3]$.

Part b.

$$f(x) - 1$$
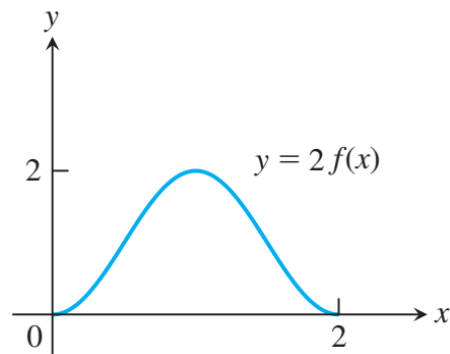
Subtracting 1 shifts the graph down by 1 unit.

The domain of $f(x) - 1$ is $[0, 2]$, and the range of $f(x) - 1$ is $[-1, 0]$.

Part c.

$$2f(x)$$

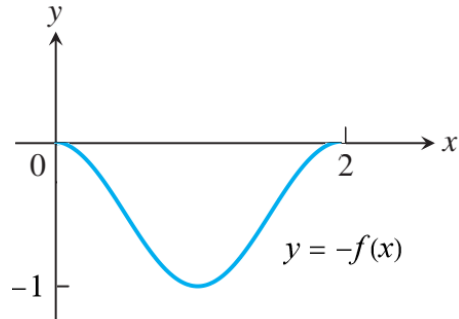Multiplying $f(x)$ by 2 vertically stretches the graph by a factor of 2.

The domain of $2f(x)$ is $[0, 2]$, and the range of $2f(x)$ is $[0, 2]$.

Part d.

$$-f(x)$$

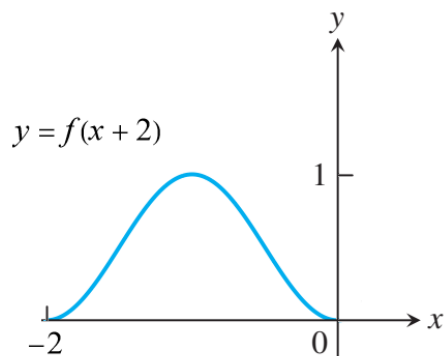Multiplying $f(x)$ by -1 reflects the graph over the x-axis.

The domain of $-f(x)$ is $[0, 2]$, and the range of $-f(x)$ is $[-1, 0]$.

Part e.

$$f(x + 2)$$

Replacing x with $x + 2$ shifts the graph to the left by 2 units.
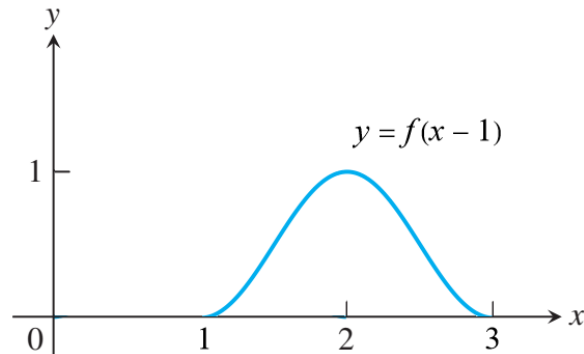

The domain of $f(x + 2)$ is $[-2, 0]$, and the range of $f(x + 2)$ is $[0, 1]$.

Part f.

$$f(x - 1)$$

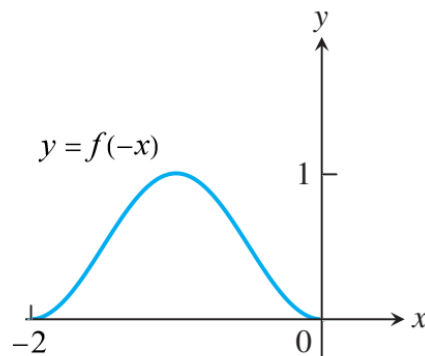Replacing x with $x - 1$ shifts the graph to the right by 1 unit.

The domain of $f(x - 1)$ is $[1, 3]$, and the range of $f(x - 1)$ is $[0, 1]$.

Part g.

$$f(-x)$$

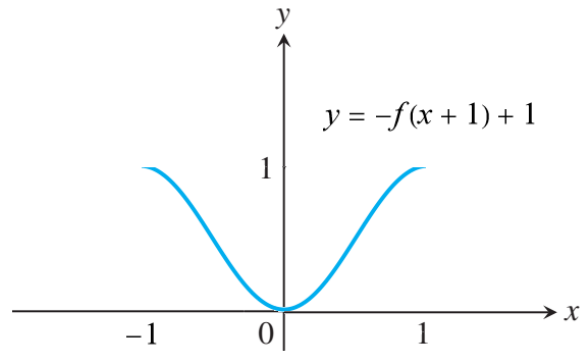Replacing x with $-x$ reflects the graph over the y-axis.

The domain of $f(-x)$ is $[-2, 0]$, and the range of $f(-x)$ is $[0, 1]$.

Part h.

$$-f(x+1)+1$$

Multiplying $f(x)$ by -1 reflects the graph over the x-axis, replacing x with $x+1$ shifts the graph to the left by 1 unit, and adding 1 shifts the graph up by 1 unit.

The domain of $-f(x+1)+1$ is $[-1, 1]$, and the range of $-f(x+1)+1$ is $[0, 1]$.